SOL HW 3.6

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Name:____________________________ Date:__________________________

Math 10/11 Enriched: Section 3.6 Basic Trigonometric Identities

$\sin(-x) = -\sin x$ $\tan x = \frac{\sin x}{\cos x}$ $\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

$\cos(-x) = \cos x$ $\sin^2 x + \cos^2 = 1$ $\sin 2x = 2\sin x \cos x$

1. ***Verify*** which of the following are trigonometric identities:

When verifying, try to use an angle that would be uncommon as an answer. ie: x = 1rad or 35rad.

a) $\tan x + \cot x = \sec x \csc x$

$\tan 25 + \cot 25° = (\sec 25)(\csc 25)$

$\tan 25° + \frac{1}{\tan 25°} \stackrel{?}{=} \left(\frac{1}{\cos 25}\right)\left(\frac{1}{\sin 25}\right)$

$= 2.610814 \quad = 2.610814$

b) $\sec^2 x + \csc^2 x = \sec^2 x \bullet \csc^2 x$

$\left(\frac{1}{\cos 30}\right)^2 + \left(\frac{1}{\sin 30}\right)^2 \stackrel{?}{=} \left(\frac{1}{\cos 30}\right)^2\left(\frac{1}{\sin 30}\right)^2$

$\frac{3}{4} + \frac{1}{4} \stackrel{?}{=} \left(\frac{3}{4}\right)\times\left(\frac{1}{4}\right)$

$1 \neq \frac{3}{16}$

Not an identity!

c) $\sec^2 x - \csc^2 x = \frac{\sec^2 x}{\csc^2 x}$

$\left(\frac{1}{\cos 30}\right)^2 - \left(\frac{1}{\sin 30}\right)^2 \stackrel{?}{=} \left(\frac{1}{\cos 30}\right)^2 \div \left(\frac{1}{\sin 30}\right)^2$

$\frac{4}{3} - \frac{4}{1} \stackrel{?}{=} \left(\frac{3}{4}\right) \div \left(\frac{1}{4}\right)$

$-\frac{8}{3} \neq 3$ Not an identity

d) $\sec^2 x + \csc^2 x = (\tan x + \cot x)^2$

If you think its an identity, try to prove it:

$\left(\frac{1}{\cos 30}\right)^2 + \left(\frac{1}{\sin 30}\right)^2 \stackrel{?}{=} \left(\frac{\sin 30}{\cos 30} + \frac{\cos 30}{\sin 30}\right)^2$

$\frac{4}{3} + 4 \stackrel{?}{=} \left(\frac{\sqrt{3}}{3} + \frac{1}{\sqrt{3}}\right)^2$

$\frac{16}{3} \stackrel{?}{=} \ldots$

$\frac{16}{3} = \frac{16}{3}$ (Yes)

$\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} = \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)^2 = \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$

$= \frac{s^2}{c^2} + 1 + 1 + \frac{c^2}{s^2} = \frac{s^2}{c^2} + \frac{c^2}{c^2} + \frac{s^2}{s^2} + \frac{c^2}{s^2} = \frac{1}{c^2} + \frac{1}{s^2}$

e) $\cos^2 x = \sin x(\csc x + \sin x)$

$\cos^2 x = \sin x \cdot \frac{1}{\sin x} + \sin^2 x$

$\cos^2 x = 1 + \sin^2 x$ (No!)

$(\cos 30)^2 = \sin 30\left(\frac{1}{\sin 30} + \sin 30\right)$

$\frac{3}{4} = \frac{1}{2}\left(2 + \frac{1}{2}\right)$

$= \frac{1}{2}\left(\frac{5}{2}\right)$

$\frac{3}{4} \neq \frac{5}{4}$ (Not an identity)

f) $\sin^2 x = \cos x(\sec x - \cos x)$

$\sin^2 x = \cos x\left(\frac{1}{\cos x} - \cos x\right)$

$= 1 - \cos^2 x$

$\sin^2 x = \sin^2 x$

Yes! Identity!

$(\sin 30)^2 = (\cos 30)\left(\frac{1}{\cos 30} - \cos 30\right)$

$\frac{1}{4} = \frac{\sqrt{3}}{2}\left(\frac{2}{\sqrt{3}} - \frac{\sqrt{3}}{2}\right)$

$= 1 - \frac{3}{4}$

$\frac{1}{4} = \frac{1}{4}$ ✓

g) $\sin x \tan x + \sec x = \frac{\sin^2 x + 1}{\cos x}$

$\sin x\left(\frac{\sin x}{\cos x}\right) + \frac{1}{\cos x} = \text{R.H.S.}$

$\frac{\sin^2 x + 1}{\cos x} = \text{R.H.S.}$ ✓

= Identity

h) $\frac{\sin x + \tan x}{\cos x + 1} = \tan x$

$\frac{\sin x + \frac{\sin x}{\cos x}}{\cos x + 1} \times \frac{\cos x}{\cos x}$

$= \frac{\sin x(\cos x + 1)}{\cos x(\cos x + 1)}$

$= \frac{\sin x}{\cos x}$ Yes, identity

2. Simplify each of the following expressions:

a) $\sin^2 x + \cos^2 x + \cot^2 x$

① $\sin^2 x + \cos^2 x = 1$

② $\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$

$1 + \cot^2 x = \csc^2 x$

$\therefore = \sin^2 x + \cos^2 x + \cot^2 x$

$= 1 + \cot^2 x$

$= \csc^2 x$ //

b) $\frac{\sin 2x}{1 + \cos 2x}$

$= \frac{2\sin x \cos x}{1 + (2\cos^2 x - 1)}$

$= \frac{2\sin x \cos x}{2\cos^2 x}$

$= \frac{2\sin x}{2\cos x}$ $\tan x$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

c) $\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x}$

$= \frac{\sin 3x \cos x}{\sin x \cos x} - \frac{\cos 3x \sin x}{\sin x \cos x}$

$\sin(3x)\cos x - \cos 3x \sin x$

$\frac{\sin(3x - x)}{\sin x \cos x}$

$\frac{\sin 2x}{\sin x \cos x}$

$\frac{2\sin x \cos x}{\ldots} = 2$

d) $\cos(a + b)\cos b + \sin(a + b)\sin b$

$\cos(\alpha - \beta)$

$= \cos(a + b - b) = \cos a$

$= \frac{\sin 3x \cos x}{\sin x \cos x} - \frac{\cos 3x \sin x}{\sin x \cos x}$ $= \frac{\sin(3x)\cos x - \cos 3x \sin x}{\sin x \cos x}$ $\frac{\sin 2x}{\sin x \cos x}$ $\frac{2\sin x \cos x}{\sin x \cos x} = \boxed{2}$	$= \cos(a+b-b) = \boxed{\cos a}$
e) $\sin\left(\frac{\pi}{3}-x\right)\cos\left(\frac{\pi}{3}+x\right)+\cos\left(\frac{\pi}{3}-x\right)\sin\left(\frac{\pi}{3}+x\right)$ α, β $\sin(\alpha+\beta)$ $= \sin\left(\frac{\pi}{3}-x+\frac{\pi}{3}+x\right) = \sin\left(\frac{2\pi}{3}\right)$ $= \boxed{\frac{\sqrt{3}}{2}}$	f) $\frac{\cos^3 x - \cos x}{\sin^3 x}$ $= \frac{-\cos x(-\cos^2 x + 1)}{\sin^3 x}$ $= \frac{-\cos x(\sin^2 x)}{\sin^3 x}$ $= \frac{-\cos x}{\sin x}$ $= \boxed{-\cot x}$

3. Suppose $0 < x < 90°$ and $2\sin^2 x + \cos^2 x = \frac{25}{16}$. What is the value of $\sin x$?

NOTE: $2\sin^2 x = \sin^2 x + \sin^2 x$

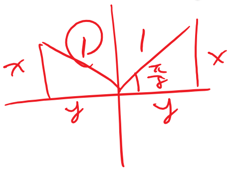
$\therefore \sin^2 x + \sin^2 x + \cos^2 x = \frac{25}{16}$

$\sin^2 x + 1 = \frac{25}{16}$

$\sin^2 x = \frac{9}{16}$

$\sin x = \pm 3/4$

4. Evaluate the following: $\sin\left(\frac{\pi}{6}\right)+\sin^2\left(\frac{\pi}{6}\right)+\cos^2\left(\frac{\pi}{6}\right)$



$\sin\frac{\pi}{8} = x$ $\quad \sin^2\frac{\pi}{8} = x^2$

$\cos\frac{7\pi}{8} = -y$ $\quad \cos^2\frac{7\pi}{8} = y^2$

5. Determine the value of $\sin^2\left(\frac{\pi}{8}\right)+\cos^2\left(\frac{3\pi}{8}\right)+\sin^2\left(\frac{5\pi}{8}\right)+\cos^2\left(\frac{7\pi}{8}\right)$

$\sin^2\left(\frac{\pi}{8}\right)+\cos^2\left(\frac{7\pi}{8}\right) \quad \cos^2\left(\frac{3\pi}{8}\right)+\sin^2\left(\frac{5\pi}{8}\right)$

$1 + 1 = 2$

6. Suppose that, for some angles "x" and "y" $\sin^2 x + \cos^2 y = \frac{3a}{2}$ and $\cos^2 x + \sin^2 y = \frac{1}{2}a^2$, determine the possible value(s) of "a".

① ADD THE TWO EQUATIONS TOGETHER:

$\sin^2 x + \cos^2 y = \frac{3a}{2}$

$\cos^2 x + \sin^2 y = \frac{1}{2}a^2$

$\sin^2 x + \cos^2 x + \cos^2 y + \sin^2 y = \frac{3a}{2} + \frac{1}{2}a^2$

$1 + 1 = \frac{3a}{2} + \frac{1}{2}a^2$

$2 = \frac{3a}{2} + \frac{1}{2}a^2$

$4 = 3a + a^2$

$0 = a^2 + 3a - 4$

$0 = (a+4)(a-1)$

$a = -4$

$a = 1$

7. What is the sum of all values of all values of "x" between 0 and 2π inclusive that satisfy the equation:

$\tan x + 1 = \sec^2 x$?

① USE AN IDENTITY TO MAKE ALL FUNCTIONS THE SAME:

$\frac{s^2}{c^2} + \frac{c^2}{c^2} = \frac{1}{c^2} \rightarrow \tan^2 x + 1 = \sec^2 x$

② MAKE A SUBSTITUTION:

$\tan x + \cancel{1} = \tan^2 x + \cancel{1}$

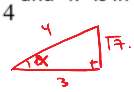
$0 = \tan^2 x - \tan x$

$0 = \tan x(\tan x - 1)$

$\tan x = 0$ | $\tan x = 1$
$x = \tan^{-1}(0)$ | $x = \tan^{-1}(1)$
$x = 0, \pi, 2\pi$ // | $x = \frac{\pi}{4}, \frac{5\pi}{4}$ //

8. If $\cos(x) = \frac{3}{4}$ and "x" is in the first quadrant, what is the value of $\sin(2x)$?

① $\cos x = \frac{3}{4}$.



② $\sin x = \frac{\sqrt{7}}{4}$ (POSITIVE).

③ $\sin 2x = \sin(x+x) = \sin x \cos x + \sin x \cos x$

$= 2\sin x \cos x$

$= 2\left(\frac{\sqrt{7}}{4}\right)\left(\frac{3}{4}\right)$

$= \frac{6\sqrt{7}}{16}$ //

9. Suppose that $\left(\sin a + \sin b = \sqrt{\frac{5}{3}}\right)^2$ and $(\cos a + \cos b = 1)^2$. What is the value of $\cos(a-b)$?

$\cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\sin^2\alpha + 2\sin\alpha\sin\beta + \sin^2\beta = \frac{5}{3}$

$+ \quad \cos^2\alpha + 2\cos\alpha\cos\beta + \cos^2\beta = 1$

$1 + 2\sin\alpha\sin\beta + 2\cos\alpha\cos\beta + 1 = \frac{8}{3}$

$(2\sin\alpha\sin\beta + 2\cos\alpha\cos\beta) \div 2$

$= \left(\frac{2}{3}\right) \div 2 \quad \frac{1}{3}$

10. In degrees, what are all ordered pairs of angles (x,y) for which both angles are between 0 and $90°$ and satisfy the equation $\sin^2 x + \sin^2 y = \sin x + \sin y$?

$\sin^2 x - \sin x = \sin y - \sin^2 y$.

$\underbrace{\sin x}_{+ve} \underbrace{(\sin x - 1)}_{-ve} = \underbrace{\sin y}_{+ve} \underbrace{(1 - \sin y)}_{+ve}$.

NOTE: BOTH SIDES CAN ONLY BE EQUAL IF THEY ARE EQUAL TO ZERO!

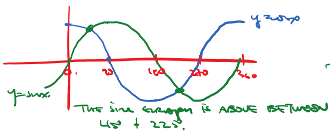
$\therefore \sin x = 0, 1$ or $\sin y = 0, 1$

$\therefore$ (x, y)
(90, 0)
(0, 90)
(0, 0)
(90, 90)

11. In degrees, what are all values of 'x" between 0 and $360°$ for which $\sin x > \sqrt{1-\sin^2 x}$?

① NOTE: $\sin x = \cos x$ ONLY AT TWO LOCATIONS:
$x = 45°$, $x = 225°$.

② DRAW BOTH GRAPHS: LOOKING FOR PART WHEN $\sin x$ IS ABOVE $\cos x$.



THE SINE GRAPH IS ABOVE BETWEEN 45° + 225°.

$\sin x > \cos x$.

$\boxed{45° \leq x < 225°}$

12. What are the degree measures of all positive angles between 0 and $90°$ which satisfy the equation:

$\sin^2 x + \cos^2 x + \tan^2 x + \cot^2 x + \sec^2 x + \csc^2 x = 31$

$1 + \tan^2\theta + \cot^2\theta + \sec^2\theta + \csc^2\theta = 31$

$\sec^2\theta + \sec^2\theta + \cot^2\theta + 1 + \csc^2\theta = 32$

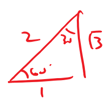
$2\sec^2\theta + 2\csc^2\theta = 32$

$\sec^2\theta + \csc^2\theta = 16$

① $s^2 + c^2 = 1$

② $1 + \cot^2\theta = \csc^2\theta$.

③ $\tan^2\theta + 1 = \sec^2\theta$.



④. $\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} = 16$

$\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta} = 16$.

$\frac{1}{16} = \cos^2\theta\sin^2\theta$.

⑤ $\frac{1}{4} = \sin\theta\cos\theta$.

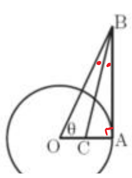
$\frac{1}{2} = 2\sin\theta\cos\theta$

$\frac{1}{2} = \sin 2\theta$.

$\boxed{2\theta = 30° \quad 2\theta = 150° \\ \theta = 15° \quad \theta = 75°}$ //

13. A circle centered at "O" has radius 1 and contains the point A. Segment AB is tangent to the circle at "A" and $\angle AOB = \theta$. If point "C" lies on $\overline{OA}$ and $\overline{BC}$ bisects $\angle ABO$, then what is the length of OC?

a) $\sec^2\theta - \tan\theta$ b) $\frac{1}{2}$ c) $\frac{\cos^2\theta}{1+\sin\theta}$ d) $\frac{1}{1+\sin\theta}$ e) $\frac{\sin\theta}{\cos^2\theta}$



① $\tan\theta = \frac{BA}{OA}$

$\boxed{\tan\theta = BA}$

SINCE OA = 1

② $\cos\theta = \frac{OA}{OB}$

$OB = \frac{OA}{\cos\theta}$

③ SINE LAW:

$\frac{\sin\theta}{OC} = \frac{\sin\angle BCO}{OB}$

$\frac{\sin\theta}{\sin\angle BCO} = \frac{OC}{OB}$.

$\frac{\sin\theta}{AC} = \frac{\sin\angle BCA}{AB}$.

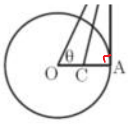
$\therefore \frac{OC}{OB} = \frac{AC}{AB}$. let $OC = x$, $AC = 1 - x$

$\frac{x}{\left(\frac{1}{\cos\theta}\right)} = \frac{1-x}{\tan\theta}$.

$x \cdot \cos\theta(\tan\theta) = 1 - x$

$x \cdot \sin\theta = 1 - x$

$\boxed{x = \frac{1}{1+\sin\theta}}$ //



since OA = 1

② $\cos\theta = \frac{OA}{OB}$

$OB = \frac{OA}{\cos\theta}$

$OB = \frac{1}{\cos\theta}$

$\frac{\sin\theta}{\sin\angle BCO} = \frac{OC}{OB}$

$\frac{\sin\theta}{AC} = \frac{\sin\angle BCA}{AB}$

$\frac{\sin\theta}{\sin\angle BCA} = \frac{AC}{AB}$

$\left(\frac{1}{\cos\theta}\right)$ $\tan\theta$

$x \cdot \cos\theta\left(\frac{1}{\cos\theta}\right) = 1 - x$

$x \cdot \sin\theta = 1 - x$

$x\sin\theta + x = 1$

$x(\sin\theta + 1) = 1$

$x = \frac{1}{1+\sin\theta}$

14. If $\cos\theta = 2\tan\theta$, solve for the numerical value of $\cos^2\theta$.

$\cos\theta = 2\frac{\sin\theta}{\cos\theta}$

$\cos^2\theta = 2\sin\theta$

$1 - \sin^2\theta = 2\sin\theta$

$0 = \sin^2\theta + 2\sin\theta + 1$

$0 = A^2 + 2A + 1$

$0 = (A+1)(A+1)$

$\therefore \sin\theta = -1$

$\sin^2\theta = 1$

Since $\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta + 1 = 1$

$\cos^2\theta = 0$

15. Simplify: $\cos\left(\frac{\pi}{6}+x\right)\cos\left(\frac{\pi}{6}-x\right) - \sin\left(\frac{\pi}{6}+x\right)\sin\left(\frac{\pi}{6}-x\right)$

① Treat $A = \frac{\pi}{6} + x$ $B = \frac{\pi}{6} - x$

using identity:

$\therefore \cos A\cos B - \sin A\sin B = \cos(A+B)$

$\therefore \cos(A+B) = \cos\left(\frac{\pi}{6} + x + \frac{\pi}{6} - x\right)$

$= \cos\frac{\pi}{3} = 0.5$

16. If $\sin x = \frac{-1}{3}$ and "x" is in quadrant 3, then what is the value of $\sin 2x$?

17. What is the value of $\sin(a+b)$ if $\sin a = \frac{-3}{5}$ and $\cos b = \frac{3}{5}$, with both "a" and "b" in the fourth quadrant.

18. Simplify the expression: $(\sin x - \cos x)^2 - (\sin x + \cos x)^2$

a) 0 b) $-\sin 2x$ c) $\sin 2x$ d) $-2\sin 2x$

$(\sin x - \cos x + \sin x + \cos x)(\sin x - \cos x - \sin x - \cos x)$

$(2\sin x)(-2\cos x)$

$= -2(2\sin x\cos x)$

$= -2\sin 2x$

19. Challenge: Evaluate and simplify the following without a calculator: $(\cos 36°)(\cos 108°)$

① $\cos 108 = \cos(180 - 72)$

$= \cos 180\cos 72 + \sin 180\sin 72$

$\cos 108° = -\cos 72$

② $\sin 72° = 2(\sin 36)(\cos 36)$

$\frac{\sin 72°}{2\sin 36°} = \cos 36°$

③ $(\cos 36)(\cos 108)$

$= \frac{(\sin 72)}{2\sin 36}(-\cos 72°)$

$= \frac{-\cos 72\sin 72}{2\sin 36}$

$= \frac{-2\sin 72\cos 72}{4\sin 36°}$

$= \frac{-\sin 144°}{4\sin 36°}$ $= \frac{-\sin 36}{4\sin 36} = -\frac{1}{4}$

④ $\sin 144 = \sin 36°$

B/C

$\sin(A) = \sin(180 - A)$